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# Networking Society: Rise and Fall

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**Abstract:** *The study of complex networks is receiving considerable attention because of its use in various socioeconomic theories. This field is growing at a rapid pace and it is underscored by researchers from physics, mathematics, biology, computer science and sociology. A well networked community consists of intense social interaction and information spreads quickly and broadly. This interaction when subjected to various parameters shows paradoxical results. Here I review a simple model which depicts the emergence of a dynamic society with three features: link formation, link deletion and search with adjustment. The model aims to make it feasible to understand how a highly dense network evolves from social interactions and to examine the statistical properties of the network. This formulation is implemented computationally to obtain average degree and clustering coefficient of the network under study. The model shows a region of coexistence of two states which is studied. The following text investigates the switch controlling the dynamic behaviour of the coexistence region and also explores the future prospects of complex networks in terms of viability of its application in the socioeconomic dimensions.*

**Keywords:** *networked society, complex network, ergodic*

## 1. INTRODUCTION

### 1.1 CONCEPT OF A NETWORKED SOCIETY

A networked society is built upon intense social interaction and the information spreads widely within the system. The environment can be either static or volatile. In a volatile environment, individuals always keep searching for fresh opportunities. It is a very important pathway of social networks which has unfolded many socio-economic phenomena. This idea can be seen through various examples one of them is finding new opportunities like jobs or investments by economic agents. It has been consistently shown by sociologists and economists [1,2] that personal relationships and peers play a commanding role in individual's life. This leads to relations among friends, relatives or neighbours in different socioeconomic dimensions. A common thesis proposed to justify this example, is in the presence of economic volatility where the quality and quantity of relationships decide one's social links and forms the basis of search.

## 2. LITERATURE REVIEW

The study of complex networks has attracted much attention [5–9], but it has been concerned mainly with simple phenomenological models reproducing some stylized facts in either stationary or non-stationary (e.g., growing) contexts. In contrast, sociological [10] and economic literature [11] has traditionally placed emphasis on understanding the main features and implications of stable social structures. Recently, however, much effort has been devoted as well to studying the dynamic forces (essentially, purposeful agent adjustment) that underlie the evolution and formation of networks in stationary social environments [12–15].

## 3. MODEL

The objective of this paper is to integrate these approaches by proposing a simple model of a society that embodies the following three features: agent interaction, search cum adjustment, and volatility (i.e. random link removal). Individuals are involved in bilateral interaction, as reflected by the prevailing network. Through occasional update, the value of some of the existing links deteriorates and is therefore lost. In contrast, the individuals also receive opportunities to search that, when successful; allow the establishment of fresh new links. Over time, this leads to an evolving social network that is always adapting to changing conditions. The model studied here is a simplification of a more complex model proposed by one of the authors [16] to understand how the network dynamics impinges on strategic behaviour.

One of the key ingredients of our model is creation of links to friends of friends, a mechanism that was introduced by Vazquez [17] in the context of growing networks. The model is also similar to that proposed in ref. 18 to explain the emergence of the small-world property [5] in social networks. In our context, we find as well that the small-world property arises when the social network is dense, but our focus is quite different. Our aim is to understand how a highly connected network may emerge from social interactions and to develop a comprehensive picture of the network's macroscopic statistical properties. In particular, we find quite nontrivial clustering properties that appear to play a key role in the dynamics. In contrast with ref. 18, our

model does not reproduce a scale-free topology, which is instead typical of growing networks (8) and static random networks with fitness driven attachment rules [19]. Rather we find single-scale networks consistent with the empirical evidence of refs. 20 and 21 on several social networks, giving support to their conjecture that link-constrained dynamics leads to single-scale distribution. Finally, among the vast recent literature on network dynamics, our work also relates to ref. 22 that found a “topological” phase transition in networks and refs. 23–25 that discuss robustness of the network with respect to removal of links or nodes and transition from highly connected to diluted networks in various contexts.

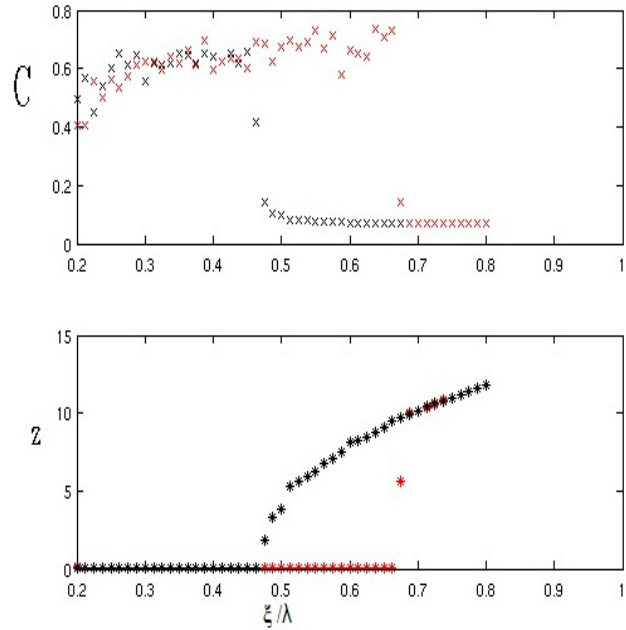
The model may be described as follows. There is a population of  $n$  agents involved in a set of bilateral interactions, as specified by the prevailing social network. This network is defined, at any given point in time  $t$ , by the (undirected) graph  $\Gamma(t) = \{N, g(t)\}$ , where  $N = \{1, 2, \dots, n\}$  is the population of nodes (or agents) and  $g(t) \in N \times N$  represents the set of links. The social interaction taking place across a link  $ij \in g$  between  $i$  and  $j$  may be conceived as, say, a collaboration that is profitable for both parties.

In any time interval  $[t, t + dt]$  any existing link  $ij \in g(t)$  vanishes with probability  $\lambda dt$ . This is interpreted as a random perturbation of the environment, or volatility for short. In addition, with independent probability  $dt$  every agent  $i$  is given the opportunity of establishing a new link with some other agent  $j$ , randomly drawn from the population. Links can also be formed through search via friends: every agent  $i$ , with a probability  $dt$ , asks one of his neighbours  $j$ , randomly chosen, to introduce him to one of  $j$ 's neighbours, say  $k$ . If  $k$  is not already a neighbour of  $i$  the link  $ik$  is established.

Naturally, nothing occurs if  $i$  has no neighbours or  $j$  has no other neighbour but  $i$ . At a heuristic level, the link formation process can be decomposed into two complementary components. On the one hand, there is the force of volatility that stamps out the value of some pre-existing links and thus, in effect, destroys them. On the other hand, there are fresh new opportunities that arise through either global search or communication with neighbors. This 2-fold interpretation of the process makes the role of information clear. The dynamics of network formation can be viewed as a continuous struggle against volatility, with the information arising on new profitable opportunities partially mediated (thus constrained) by the existing network. In the stationary state agents' constant search must compensate volatility.

#### 4. RESULTS

The results have been stimulated using MATLAB and have been verified to be similar to those calculated mathematically.



**Fig. 1.** Average degree  $z$  (lower) and clustering coefficient (upper) from computational simulations with  $\eta = 0.001$  and  $\lambda = 0.1$  for population size  $n = 100$ . The process was performed as follows. At each time step, with a probability  $\eta$ , a long range link was added between two nodes taken at random. The probability of  $\zeta$ , a local search process was done randomly. At the end, a set of randomly selected links are removed with a probability of  $\lambda$ . This process is carried out for  $n$  such time steps. Each of the points in the plot is obtained by taking averages over the nodes and over time.

The three rates ( $\lambda$ ,  $\eta$ , and  $\zeta$ ) are the parameters of our model, but one of them can be eliminated by an appropriate time rescaling. We are interested in the properties of the network  $g(t)$  in the stationary state as  $t$  tends to infinity. Relevant magnitudes in this respect are the density of the network and its clustering. Network density at any  $t$  is measured by the average node degree  $z(t)$ , where the degree  $z_i(t)$  of a node  $i$  is defined by the number of neighbors it has. On the other hand, network clustering  $C(t)$  is obtained by averaging the clustering coefficient  $C_i(t)$  of all nodes  $i$ , which is the fraction of pairs of neighbours of  $i$  who are also neighbors among themselves. Although random networks have  $C_i \sim \frac{1}{n}$ , social networks typically have a clustering coefficient (5) bounded above zero.

Fig. 1 shows what happens in a computer experiment where the local search rate is  $\zeta$  first increased and then decreased very slowly. For small  $\zeta$ , network growth is limited by the global search process that proceeds at rate  $\eta$ . Clusters of more than two nodes are rare, and when they form local search quickly saturates the possibilities of forming new links. Suddenly, at a critical value around  $0.6$  ( $\zeta/\lambda$ ), a giant component connecting a finite fraction of the nodes emerges. The average degree  $z$  indeed jumps abruptly

around  $0.6 (\xi / \lambda)$ . The network becomes more and more densely connected as increases further. But when decreases, we observe that the giant component remains stable also beyond the transition point  $\{0.6 (\xi / \lambda)\}$ . Only at this point does the network lose stability and the population gets back to an unconnected state. So, here we check the switch

because of which this bi-stability may be subject to change. A lot of experiments have been carried out to see how many jumps happen in the bi-stability region. This experiment is carried out for different values of  $k (\xi / \lambda)$  around  $0.5 - 0.6$  for average degree ( $z$ ) of the system.

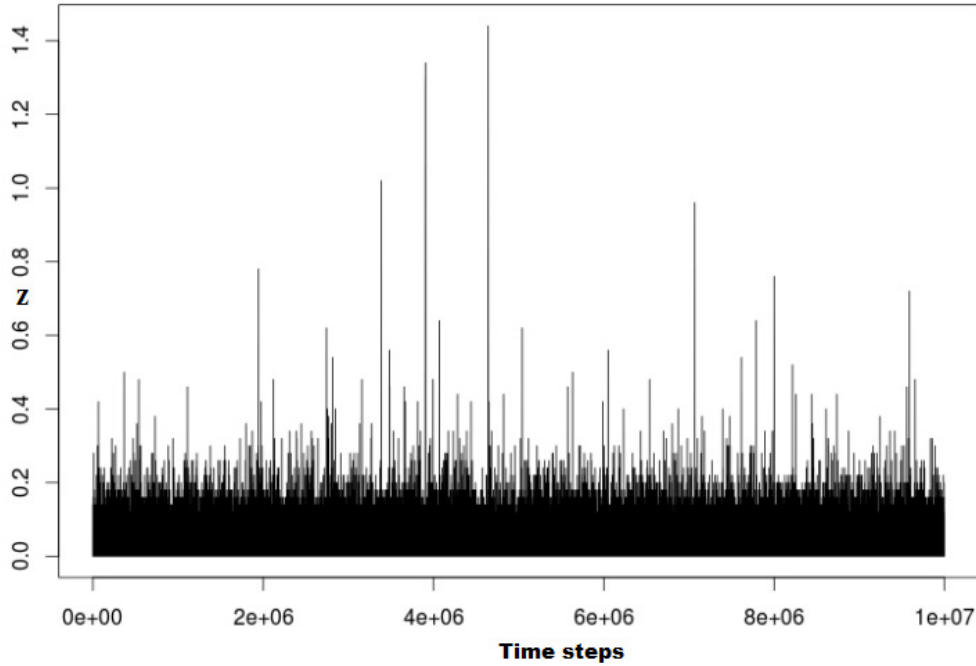


Fig. 2. Graph plotted for average degree ( $z$ ) versus 10,000,000 time steps to observe jumps in for a specific value of  $k(\xi/\lambda) = 0.475$

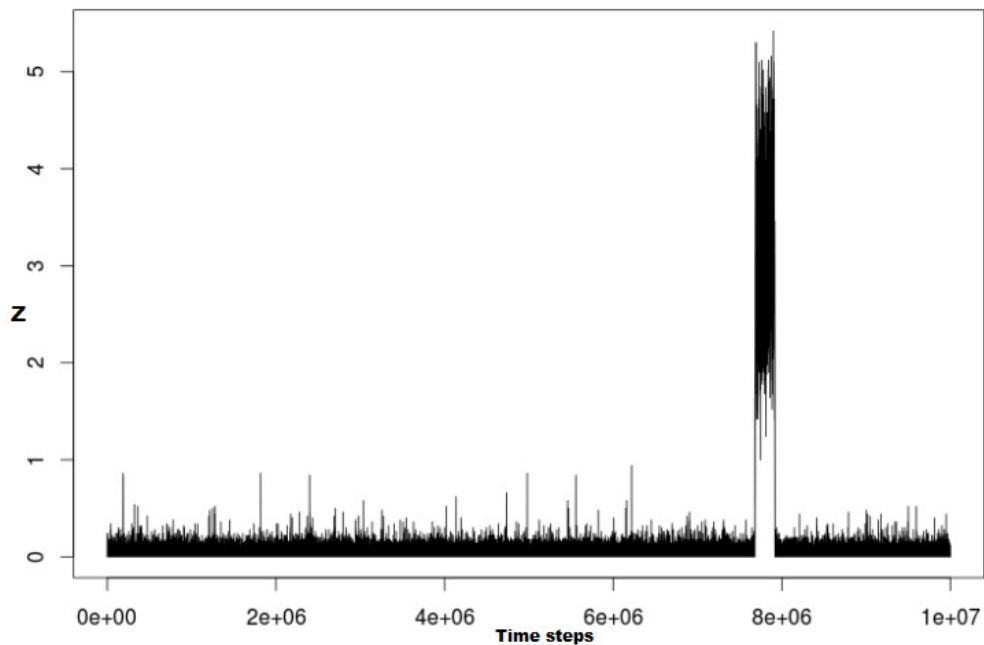
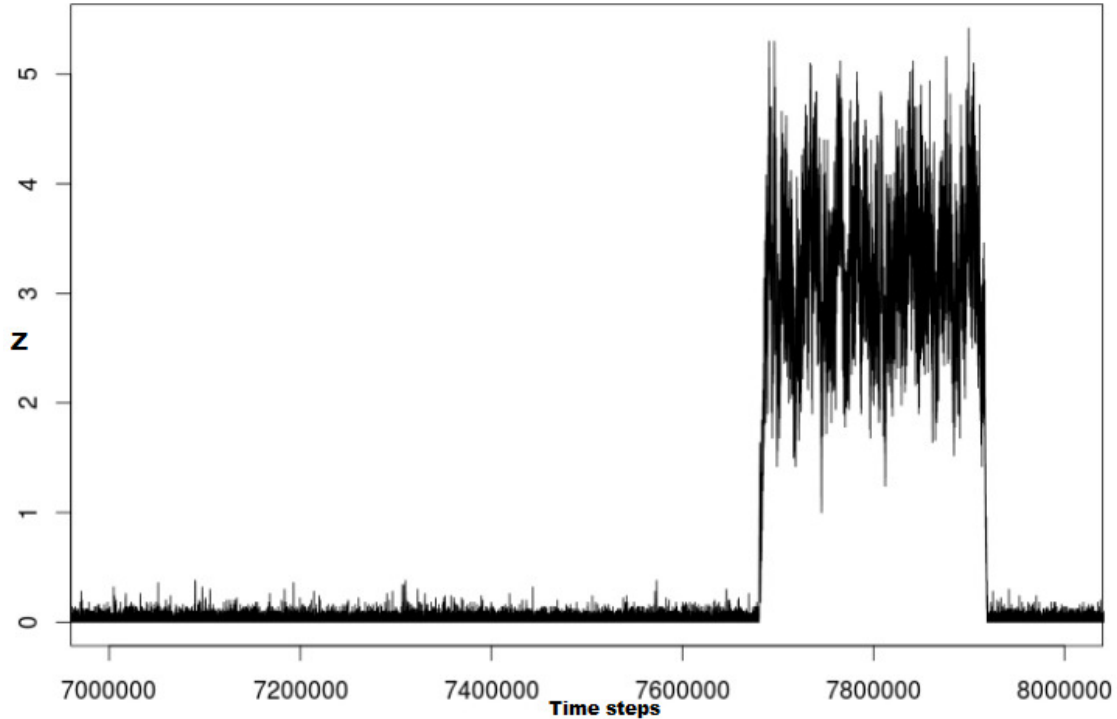


Fig. 3. Graph plotted for average degree ( $z$ ) versus 10,000,000 time steps to observe jumps in for a specific value of  $k(\xi/\lambda) = 0.4875$



**Fig. 4. Zoomed Version: Graph plotted for average degree ( $z$ ) versus 10,000,000 time steps to observe jumps in for a specific value of  $k(\xi/\lambda) = 0.4875$**

There is a whole interval where both a dense-network phase and one with a nearly empty network coexist. The coexistence region shrinks as  $\eta$  increases and it disappears for  $\eta > 0.03$ . This behaviour attains already for moderately small  $n$ , even though in this case finite size effects are strong. The average clustering coefficient  $C$  shows a nontrivial behaviour. In the unconnected phase,  $C$  increases with as expected. In this phase,  $C$  is close to one because the expansion of the network is mostly carried out through global search, and local search quickly saturates all possibilities of new connections. On the other hand, in the dense-network phase,  $C$  takes relatively small values. This makes local search very effective. Remarkably, we find that  $C$  decreases with in this phase, which is rather counterintuitive: by increasing the rate at which bonds between neighbours form through local search, the density  $C$  of these bonds decreases.

The stability of the dense network phase in the coexistence region confers resilience to the system. It implies that a dense network is robust with respect to deteriorating conditions (higher or smaller) and it may resist even under conditions in which a stable dense network would not form. In fact, similar behaviour is found, fixing and, as a function of the volatility rate. The system behaviour observed in Fig. 1 is typical of first-order phase transitions and is remarkably similar to the rise of hysteresis in physics, a phenomenon that has its origins in the ergodicity breakdown. Even if, in principle, the process is ergodic, because all configurations

can be reached from any other configuration, when  $n$  is large the configuration space gets broken into different ergodic components. Transitions across the boundaries of these components require large deviations that occur only with a probability that is exponentially small in  $n$  (they require fluctuations out of equilibrium in a collection of local neighbourhoods whose number is of order  $n$ ; see below). The occurrence of phase coexistence in our model is also intuitive and has many analogies with that of a real fluid: the local process ( $\xi$ ) mimics short-range attractive interaction, whereas the  $\lambda$  and  $\eta$  processes capture the effects of temperature and random collisions. Increasing is analogous to compressing the fluid (reducing the volume), which increases the chances that two molecules enter into the range of mutual interaction. An important difference is that interaction is long ranged in our model, which, as discussed in ref. 27, makes it impossible to have bubbles of one phase into the other: the system is either all in one or in the other phase.

## 5. CONCLUSIONS

The transitions have been studied and few peaks are observed in the graphs plotted for average degree ( $z$ ) and  $n$  time steps which indicate that within the region there are crests and troughs. These crests and troughs are indicative of the fact that it is not a clean transition from one state to another but it is subject to volatility. Strictly speaking, transitions between the two components will occur, but one typically has to wait astronomically large times.

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